

# SUPPLEMENT TO CHAPTER 9 OF REACTOR PHYSICS FUNDAMENTALS

This supplement provides mathematical background and detail not in the text. It also reviews the text material from different points of view. You should be familiar with the text material before studying this supplement.

## SUBCRITICAL REACTOR OPERATION

Review of Principles

Data Table: Delayed and Photo-neutron Groups

Neutron Sources

Source Neutron Amplification and Amplification Gain  $\frac{1}{1 - k}$

Mathematical Model and Examples

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## SUBCRITICAL REACTOR OPERATION

When you review chapter 9, do not let the algebra hide the simple ideas. Adding neutron absorbing material to the critical reactor core makes  $k < 1$ , so the **self sustaining** chain reaction stops, but **fission does not stop** ( $k > 0$ ). Any neutron that enters the fuel has a chance of initiating a fission, producing additional neutrons.

1. At power, there are many fission neutrons, so the *source neutrons*, by comparison, are not significant. Sources do affect a subcritical core.
2. Not counting neutrons from induced fission, there are three **sources** of neutrons in a reactor:
  - (a) **Spontaneous Fission neutrons:**  
These neutrons (mainly from U-238) contribute a flux of close to  $1 \text{ n-cm}/(\text{cm}^3 \text{ s})$ ; about  $10^{-12}\%$  of full power.
  - (b) **Photo-neutrons:**  
High energy gamma rays ( $E > 2.22 \text{ MeV}$ ) accompany  $\beta^-$  decay for some fission products. These gamma rays interact with the heavy water, knocking out neutrons. [For  ${}^2_1\text{H}(\gamma, n){}_1^1\text{H}$  see section 2.9, pg 31]. At power, this source is less than 10% of the small delayed neutron flux. (Actual strength depends on the operating history.)
  - (c) **Delayed Neutrons:**  
It may be convenient to think of the delayed neutron precursors as sources immediately after shutdown. Delayed neutrons are a small portion of the fission neutrons at power, but significant neutron production from precursor decay continues for 5 to 10 minutes after shutdown. At power, and at the moment of shutdown, they contribute about 0.5% of the overall thermal flux.
3. When a thermal neutron enters the fuel, no matter where it comes from, and no matter how much absorber there is in the core, it *may* cause a fission, creating additional neutrons.
4. Because of fission, *there are always more neutrons in the subcritical core than just the source neutrons*. The subcritical core ( $k < 1$ ) cannot support a self-sustaining chain reaction, but fission **amplifies** any source neutrons.

5. With lots of absorber in the moderator, (a deeply subcritical reactor) the chance of a fission causing additional fissions is not very big. A few fissions are triggered by source neutrons. Most chain reactions end almost as soon as they start. Few fission neutrons are produced, so amplification is small.
6. If the reactor is almost critical, many fissions occur. They are not self sustaining and each chain reaction eventually dies out, but some continue for many cycles. There are a lot of fission neutrons, so amplification is large.
7. The **observed** flux in a subcritical reactor includes both source neutrons and fission neutrons generated by the sources. Fission neutrons always outnumber source neutrons, unless the reactor is deeply subcritical.
8. When absorber is removed from a subcritical reactor, (leaving it subcritical), more source neutrons find their way into the fuel and amplification by the core increases. This increases the observed flux.
9. For a given  $+\Delta k$  addition, the size of the observed flux increase is bigger the nearer the reactor is to critical, because the amplification increases.

**The reactor core behaves like a neutron amplifier, multiplying the source neutrons by contributing additional fission neutrons. The multiplication factor (amplifier gain) is  $[1/(1-k)]$ . (Ch 9, section 9.4).**

10. The time it takes for the subcritical power to settle at a new level depends on how many generations it takes for a source initiated chain reaction to die out. In a deeply subcritical core all chain reactions end in a few cycles, and the power rise ends quickly. It takes longer when the reactor is nearly critical. With less absorber, fission continues for many cycles before it stops.  
(Ch 9, section 9.5)

**The time to get within 1% of the final steady state power can be estimated by  $k^N < 0.01$ . N is the number of neutron cycles:  $N = t/\lambda$  where  $\lambda \approx 0.2$  s.**

(The second longest lived delayed neutrons build to a new equilibrium level in a little less than 0.2 s. e.g.  $\lambda = 0.995 \times 0.001 \text{ s} + 0.005 \times 32 \text{ s} \approx 0.16 \text{ s}$ )

## Behaviour of Subcritical Core When +Δk is added

### Example

The following chart calculates the neutron population from one prompt generation to the next with a constant 1024 source neutrons added each cycle.

Source Strength 1024 n/cycle	Reproduction Factor k = 0.5 (Δk = -500 mk)			Add +100 mk Now k = 0.6 ↓			Source Still 1024 More neutrons survive k = 0.6 (Δk = -400mk)			
	TIME INCREASES TO THE RIGHT →	This Generation	Next Generation	0 <sup>n</sup>	1.	2.	3.	1.	2.	3.
										1024
									1024	614
								1024	614	369
				1024	614	369				221
		1024 source		512	307	184				111
added now →	1024 source	512		256	154	92				55
from before →	512	256		128	77	46				28
2nd last →	256	128		64	38	23				14
3rd last →	128	64		32	19	12				7
4th last →	64	32		16	10	6				4
5th last →	32	16		8	5	3				2
6th last →	16	8		4	2	1				1
7th last →	8	4		2	1	1				0
8th last →	4	2		1	1	0				
9th last →	2	1		0						
tenth last →	1	0								
None survive from more than 10 generations ago →	0			No Change Yet						
<b>TOTALS</b> →	2047 1024 source 1024 fission	2047 1024 source 1024 fission		2047 1024 s 1024 f	2252 1024 s 1228 f	2375 1024 s 1351 f				2450 1024 s 1426 f
The steady power level is given by $1024 \times [1/(1-k)]$ .										
Initially the multiplication factor $1/(1-k) = 2$ giving $1024 \times 2 = 2048$ .										
Increase from one generation to the next after k increases (+ 205) (+ 123) (+ 75)										

Notice that in any generation, the neutron population (total at the bottom) includes neutrons initiated by fissions that occurred as long as 10 generations ago.

Continuation of Table from Previous Page.

Numbers in the bottom part of the table have values that are still influenced by the previous reactivity level. This influence gradually disappears and the new power level (subcritical) is reached.

Subsequent Power Rise when + 100 mk is added to give $\Delta k = -400$ mk (from -500 mk)							
4.	5.	6.	7.	8.	9.	10.	Finally
							1024
					1024		614
				1024	614		369
			1024	614	369		221
		1024	614	369	221		133
	1024	614	369	221	133		80
1024	614	369	221	133	80		48
614	369	221	133	80	48		29
369	221	133	80	48	29		17
221	133	80	48	29	17		10
133	80	48	29	17	10		6
66	40	24	14	9	5		4
33	20	12	7	4	3		2
17	10	6	4	2	1		1
8	5	3	2	1	1		1
4	2	2	1	1	0		0
2	1	1	0				
<b>TOTALS→</b>	2519	2537	2546	2550	2553		2560
	1024 s		1024 s				
1467	1495 f	1513 f	1522 f	1526 f	1529 f		1536 f
Multiplication Factor $1/(1-k)$ is now $1/0.4 = 2.5$ so power will settle at $1024 \times 2.5 = 2560$ . (Numbers in the columns have been rounded to the nearest neutron).							
The influence of previous generations is nearly limited by $1024 \times k^N < 1$ (each generation is 0.001 s) The time to settle is actually much longer as delayed neutrons must be taken into account.							
(+ 41)	(+ 28)	(+ 18)	(+ 9)	(+ 4)	(+ 3)		

## Spontaneous Fission Source Flux Estimate

Here is an estimate of the number of neutrons emitted into a fresh core by spontaneous fission, and the neutron flux that results from this process. Spontaneous fission is a continuous, random process governed by the U-238 half life for spontaneous fission ( $T_{\text{s.f.}} = 8 \times 10^{15}$  years) and by the amount of U-238, and is not affected by power level, temperature etc.

A freshly fuelled reactor with 380 channels of 12 bundles, each with 18.9 kg of natural uranium (99.28% U-238 and 0.72% U-235) holds 86 tonnes of U-238 and 620 kg of U-235. (You can check the arithmetic).

There are 6.9 spontaneous fissions/s from each kg of U-238, for a total of about 590,000/s for the whole core. (The relatively small amount of U-235 adds an insignificant 0.3 spontaneous fissions/kg < 200 fissions/s for the whole core).

Each U-238 spontaneous fission produces, lets guess, 2.5 neutrons. Each neutron survives, on average, for about 1 ms (0.0009 s is the prompt neutron lifetime), so the number of neutrons in the whole core from this source at any given moment is 590,000 (fissions/s)  $\times$  2.5 (neutrons/fission)  $\times$  0.0009 s = 1300 neutrons. Another way to think of this is that 1300 neutrons are born and die in each millisecond.

The entire core volume is the volume of a cylinder about 7 m across and 6 m long, so  $V = \pi r^2 l \approx 231 \text{ m}^3$ . Combining the number of neutrons and the core volume gives an estimate of the average neutron density:  $1300/231 \approx 6 \text{ n/m}^3 = 6 \times 10^{-6} \text{ neutrons per cm}^3$ .

The average thermal flux from these neutrons is  $\phi = nv$   
where  $v = 2200 \text{ m/s} = 220,000 \text{ cm/s}$ , giving  $\phi = 1.3 \text{ n-cm/cm}^3\text{-s}$

If the fresh core is heavily poisoned with  $k \approx 0.5$ , the observed flux, from  $\phi = \phi_{\text{source}}/(1-k)$  is  $2 \times \phi_{\text{source}} \approx 3 \text{ n-cm/cm}^3\text{-s}$ . This is a typical value for estimates for a freshly fuelled CANDU core.

## Neutron Sources

The factor  $P_{source}$  in the equation  $P_{observed} = \left(\frac{1}{1-k}\right) P_{source}$  drives the process of subcritical multiplication. If there was no source there would be no observed flux, but when there is a source there is always an observed flux that is larger than the source.

The flux in the core that can be attributed to spontaneous fissions is conveniently taken to be about 1 n-cm/cm<sup>3</sup>-s at any moment. (See the estimate in the box opposite). This is about 10<sup>-14</sup> of the full power flux<sup>1</sup>. Spontaneous fission occurs at this rate whenever U-238 is present (always) so these neutrons are always present in the core, independent of power level, temperature, pressure, etc. The neutron concentration and flux from spontaneous fission never changes. That is why we call it a source.

The observed flux includes the source neutrons, the neutrons created when the source neutrons cause fissions, and from fissions caused by fission neutrons. In other words, when we “shut the reactor down”, neutron multiplication occurs even though introduction of a single pulse of neutrons will not result in a self sustaining chain reaction. [ $k \ll 1$  but  $k > 0$ ].

Chapter 9 adds the source neutrons now present ( $\phi_{source}$ ) to the fission neutrons created by the previous generation of source neutrons ( $k \phi_{source}$ ) to the fission neutrons created by the fission neutrons created by the source neutrons three generations ago  $k \times (k \phi_{source})$  to the fission neutron created by the etc. etc. to get:

$$\begin{aligned}\phi_{observed} &= \phi_{source} + k\phi_{source} + k \times (k\phi_{source}) + \text{etc.} \\ \phi_{observed} &= \phi_{source} (1 + k + k^2 + k^3 + \dots)\end{aligned}$$

This adds up<sup>2</sup>, if  $k^n$  is sufficiently small, to give:  $\phi_{observed} = \phi_{source}/(1-k)$

<sup>1</sup> 10<sup>14</sup> n-cm<sup>-2</sup> s<sup>-1</sup> is a typical CANDU full power “effective” flux in the fuel. The full power average flux in the moderator, (away from absorbers), is about 2 × 10<sup>14</sup> n-cm<sup>-2</sup> s<sup>-1</sup>

<sup>2</sup> Add like this:

Multiply this by k and place it below, as follows:

$$\begin{array}{r} \phi_{observed} = \phi_{source} (1 + k + k^2 + k^3 + k^4 + \dots) \\ k \times \phi_{observed} = \phi_{source} (k + k^2 + k^3 + k^4 + k^5 \dots) \\ \hline (\phi_{observed} - k \times \phi_{observed}) = \phi_{source} \times (1) \text{ so } \phi_{observed} \times (1-k) = \phi_{source} \\ \text{giving } \phi_{observed} = \phi_{source}/(1-k) \end{array}$$

## Delayed Neutron and Photo-neutron Data for Equilibrium Fuel

GROUP	DELAYED NEUTRON FRACTION % OF TOTAL OF $\beta = 0.54\%$	HALF LIFE weighted av = 8.4 s mean lifetime 12.2 s
6	3.5%	0.2s
5	14.0%	0.5s
4	39.0%	2.2s
3	19.0%	5.7s
2	21.0%	22.0s
1	3.5%	54.2 s

	PHOTONEUTRON FRACTION % OF TOTAL OF $\beta_{PN} \approx 0.033\%$ (0.000853/fission)	HALF LIFE
15	64.6%	2.5s
14	20.3%	41.0s
13	7.0%	2.40m
12	3.3%	7.70m
11	2.1%	27.0m
10	2.3%	1.65h
9	0.3%	4.41h
8	0.1%	53.04h
7	0.05%	12815d

Note: The longest lived photo-neutron source is due mainly to the decay of the fission product Ba-140, for which about 4% of  $\beta$  decays yield  $\gamma$ -rays with  $E > 2.23$  MeV.

## Fresh Fuel Delayed Neutron Data

	DELAYED NEUTRON FRACTION $\beta = 0.7\%$	HALF LIFE Weighted Average = 8.8 s Mean Lifetime = 12.7 s
<b>GROUP</b>	<b>Fraction of <math>\beta</math></b>	
6	2.6%	0.2 s
5	12.8%	0.5 s
4	40.7%	2.2 s
3	18.8%	6.0 s
2	21.3%	21.9 s
1	3.8%	54.6 s

Consider, now, the delayed neutron and photo-neutron “sources”. These neutrons appear many generations after the fission that causes them because they are an indirect result of fission, occurring after a fission product decay.

*Delayed neutrons* are emitted during the ( $\beta^-,\gamma$ ) decay of certain fission products. Sometimes the daughter nucleus in a decay sequence is created in an energetic state and ejects a neutron. e.g. Br-87 is a fission product that decays to Kr-87 with a half life  $T_{1/2} = 55.9$  s. In some cases the Kr-87 nucleus is produced in a high energy state that immediately releases a neutron (becoming Kr-86).

There are dozens of such decays. In the Chart of Nuclides there are almost 100 precursors, marked with a small (*n*), but their effect is accurately mimicked by 6 groups of delayed neutrons with half lives ranging between 0.2 s and 54 s, given in the table opposite. Conveniently, but less accurately, the delayed neutrons can be represented by a single group with a decay constant,  $\lambda$ , of  $0.076 \text{ s}^{-1}$  ( $T_{1/2} = 9.1$  s). The sequence given above for Br-87 is the main contribution to the longest lived (55 s) group of delayed neutrons.

*Photo-neutrons* are essentially delayed neutrons with long half lives. In the CANDU core they are produced when energetic gamma rays ( $E > 2.2$  MeV) interact with deuterium nuclei in heavy water. The 2.2 MeV energy threshold for this reaction is the binding energy of deuterium.

Many fission products decay emitting energetic gamma rays, but their effect can be mimicked by the 9 photo-neutrons groups listed opposite, with half lives ranging from 2.5 seconds to 12.8 days.

If full power flux is exactly  $10^{14}$  n-cm/cm<sup>3</sup>-s:

Prompt neutrons from induced fission:	99,433,999,999,999
Delayed neutrons	534,000,000,000
Photo-neutrons	32,000,000,000
Spontaneous fission	<u>1</u>
Total	100,000,000,000,000

This numerical illustration is not meant to be exact. In a core at steady power, for equilibrium fuel burnup, the delayed neutrons fraction is about 0.5% of the flux, the photo-neutron fraction is about 0.03% of the flux and the spontaneous fission source is an insignificant  $10^{-12}$ % of the full power flux.

The delayed neutrons, with their short half lives, drop to insignificant levels 5 to 10 minutes after shutdown. During this short time they are present in excess and behave like a fairly large, decreasing source that prevents the reactor power from dropping quickly below the 5% to 10% range.

The fundamental characteristic of a neutron source, illustrated by spontaneous fission, is that it produces a *constant supply* of neutrons, independent of the power level. How is it that we can call photo-neutrons neutron sources when their equilibrium concentration is proportional to the rate of fission? We say this because the concentration changes *gradually* so that the concentration does not depend on the present power, but on the previous power.

In particular, if you shut the reactor down for two weeks, the delayed and photo-neutrons drop back to equilibrium fractions of the subcritical flux, except for the long lived ( $T_{1/2} = 12.8$  day) component of the photo-neutron flux. After 2 weeks (14 days) it has dropped to a little below half of what it was at shut down. Short of waiting for about a year<sup>3</sup> there is nothing that can make these neutrons disappear. They behave like a *gradually decreasing source*, fixed by the reactor power in the weeks before shut down, unrelated to the present fission rate.

### Responsiveness of the Subcritical Core

A deeply subcritical reactor and an almost critical reactor behave differently when positive reactivity is inserted. The following graph of the amplification factor shows why.

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<sup>3</sup> It takes 24 half lives ( $24 \times 12.8$  days = 307 days) for this component to drop back to the strength we attributed to spontaneous fission source. CANDU behaviour points to another group with ( $T_{1/2} \approx 94$  days)

When deeply subcritical, ( $k$  very small), the subcritical multiplication factor,  $[1/(1-k)]$ , is nearly equal to 1, and a large fraction of the observed flux is the source. Changes in reactivity in such a core have a relatively small effect on the observed flux.

When the core is nearly critical, ( $k \approx 1$ ), the subcritical multiplication factor,  $[1/(1-k)]$ , is large and most of the observed flux is fission flux. Changes in reactivity produce large changes, almost independent of the source flux. The core behaves more like a critical core than like a deeply subcritical core.

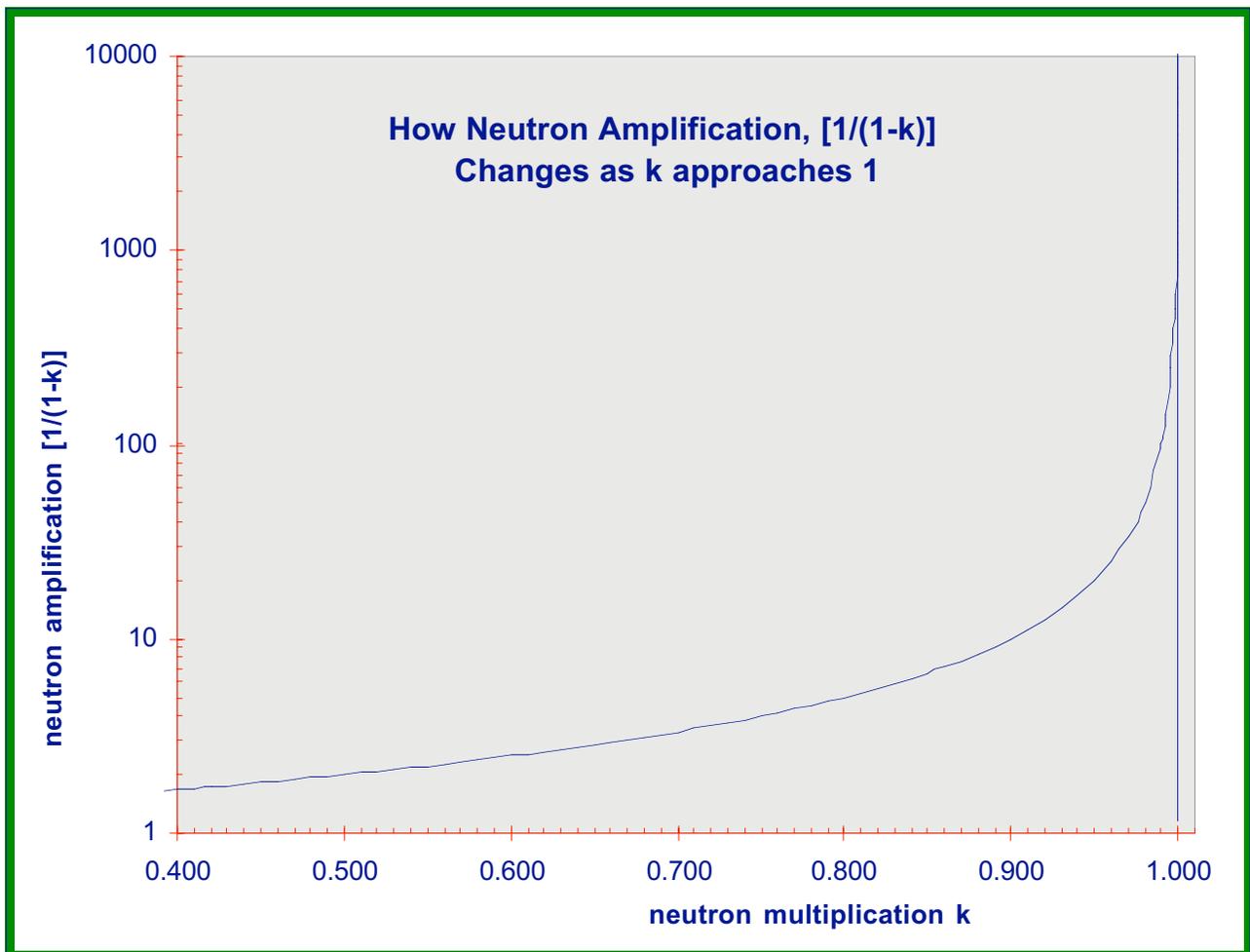


Figure 1 Neutron Amplification in the Subcritical Core

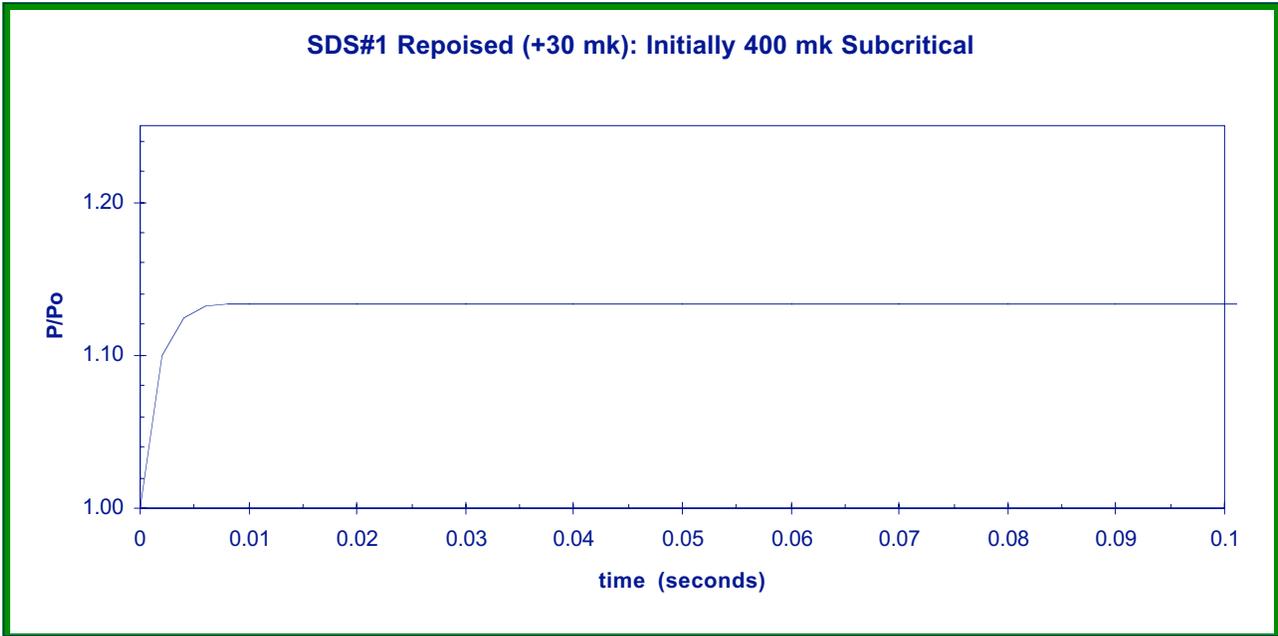


Figure 2

∞ In a deeply subcritical core, a very large reactivity insertion produces an almost unnoticeable power rise. Power stabilizes almost immediately. Notice how quickly the rise occurs. (A step insertion is assumed).

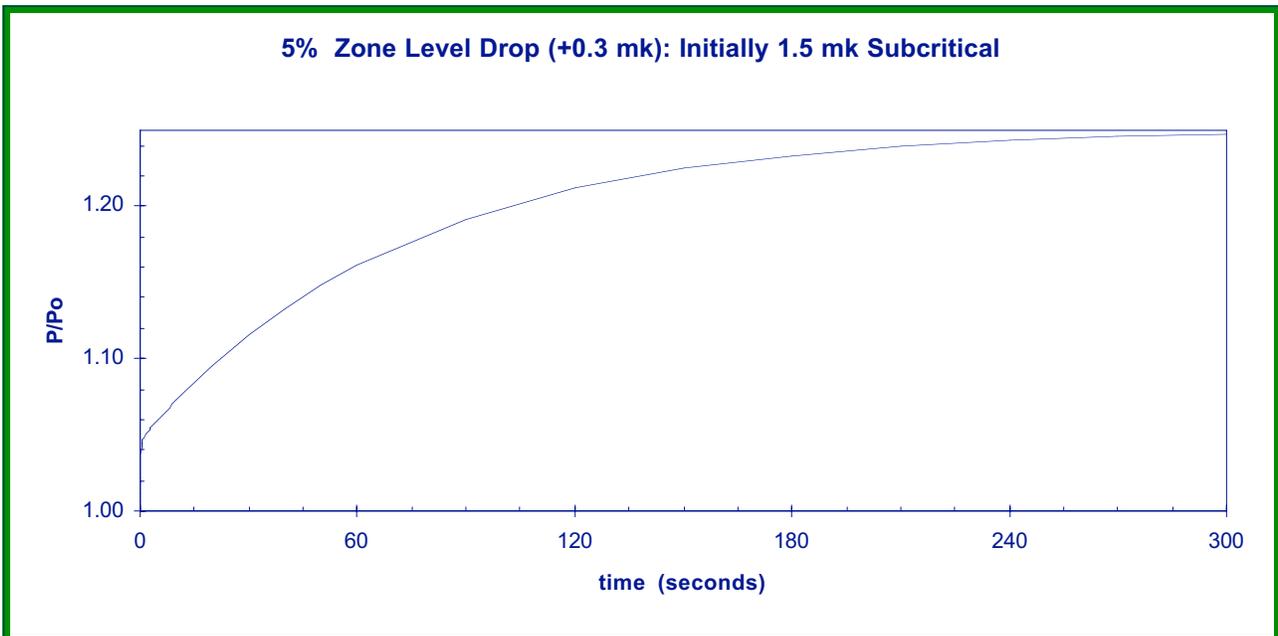


Figure 3

∞ Power rise in an almost critical core is large, even for a small reactivity insertion. Power rises for several minutes after the insertion.

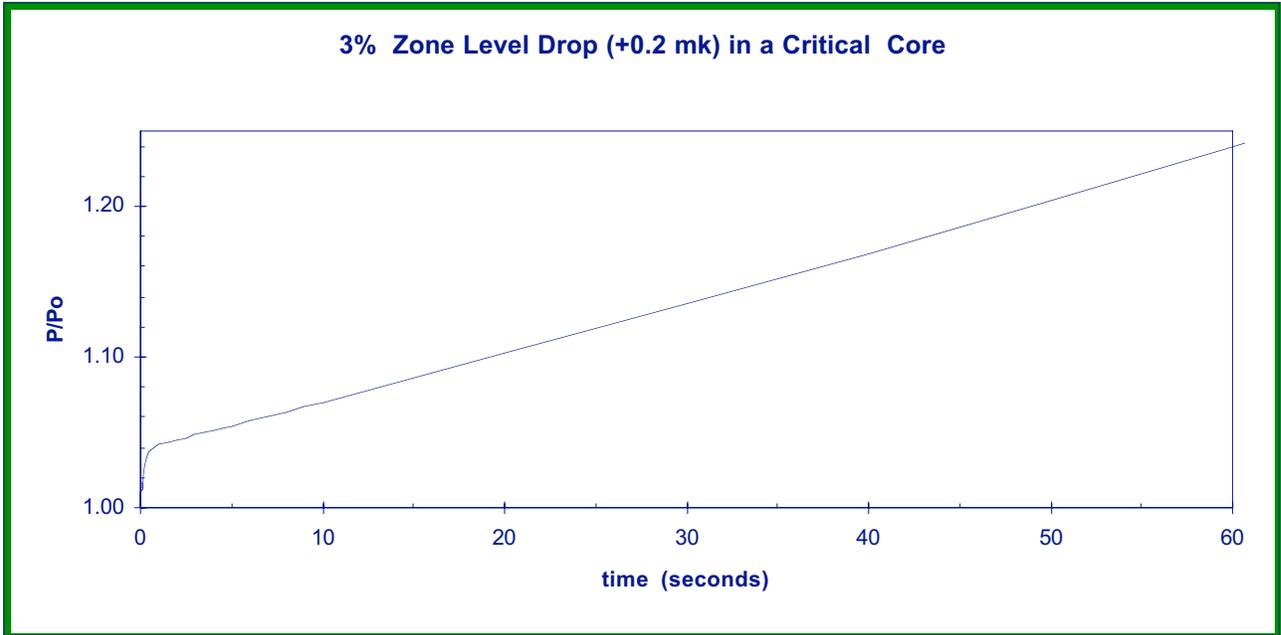


Figure 4

∞ Compare this with the first 60 seconds of Figure 3. Responses of an almost critical reactor and a critical reactor are similar. There is no easy way to distinguish the immediate power response in the two operating states.

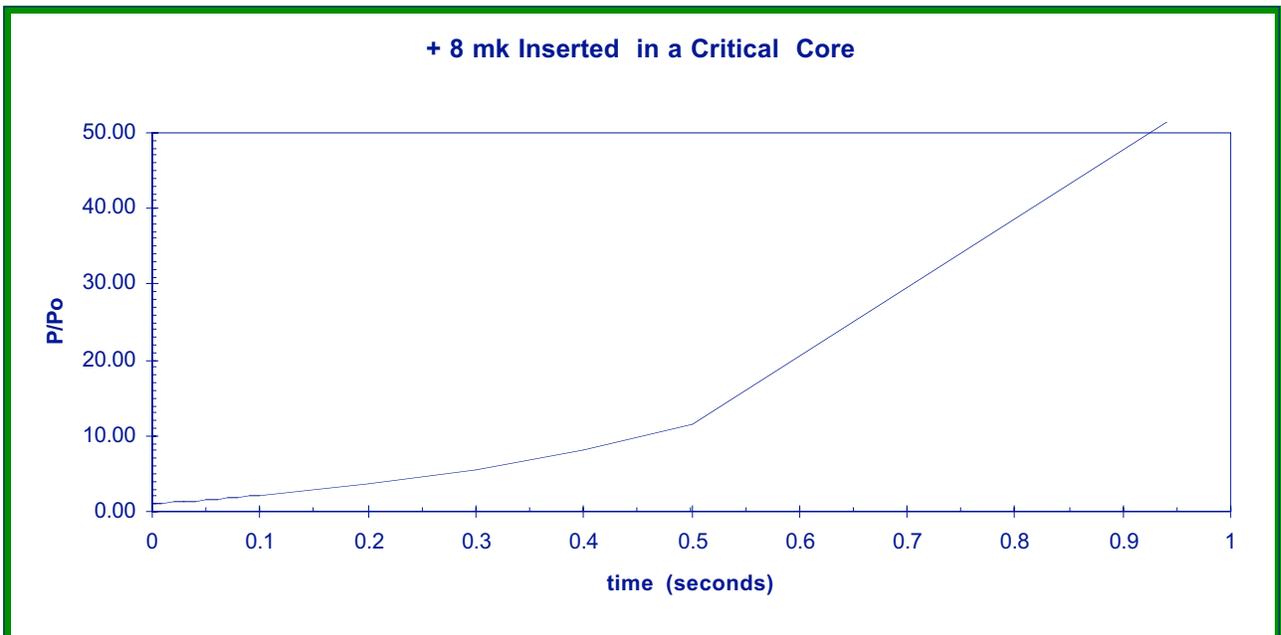


Figure 5

∞ This speaks for itself. It is a sobering thought that inserting enough positive reactivity to make the reactor super prompt critical can result in a power increase of several thousand per cent in seconds.

When the core is taken slightly supercritical, the response depends on the self sustaining chain reaction and is de-coupled from the source. There is no sudden, dramatic change at  $k = 1$ . As  $k$  increases and approaches  $k = 1$  the core response becomes less and less dependent on the source neutrons. The almost critical core is sensitive to a small reactivity increase, and the resulting power increases can take several minutes to stabilize. Comparison of figures 2. and 3. shows the similarity between the almost critical core and the slightly supercritical core in the first 30 s or so after a reactivity insertion.

Manual start up of a reactor must be done with great care. If monitoring is not adequate, a careless reactivity insertion could cause a “premature criticality event”, with the risk of an uncontrolled power excursion. Figure 4 shows a worst case response. In less than 1 s the power has gone out the top of the graph; a 5000% increase. Starting at, say  $10^{-4}$  the power reaches 100% in just over 2.5 seconds, and rises to 1000% in the next second.

The equations used to model power rise in these pictures are from a set of lecture notes LECTURES ON REACTOR KINEMATICS by Ben Rouben (AECL) and Gabe Balog (OH), delivered to Ontario Hydro safety department staff in November 1995. These notes give the solutions for the two group differential equations for various operating states. For the subcritical reactor, the differential equation introduced in the Chapter 8 supplement must include a constant source term in addition to the  $\lambda C$  term.

The solutions are shown on the next couple of pages. The parameter used for reactivity is slightly different than in our text. The difference,  $(k-1)$ , between the neutron multiplication factor,  $k$ , and its value for a critical core,  $k = 1$ , is a measure of how subcritical the core is. It is negative for  $k < 1$ . The textbook measure of *reactivity*<sup>4</sup>,  $\rho$ , is the fractional difference between  $k$  and 1,  $\rho = \frac{k-1}{k} = 1 - \frac{1}{k}$ . ( $\rho$  is used in the following equations in place of  $\Delta k$ )

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<sup>4</sup> It should be pointed out that  $\rho$  and  $(k-1)$  are nearly equal when  $k \approx 1$ . (e.g. if  $k = 0.990$ ,  $k-1 = -0.010 = -10$  mk and  $\rho = -0.010/0.990 = -10.1$  mk). On start up, the core may be deeply subcritical with  $\rho$  and  $(k-1)$  quite different. (e.g. if  $k = 0.600$ ,  $k-1 = -0.400 = -400$  mk and  $\rho = -0.4/0.6 = -667$  mk, 267 mk lower.)

## Power Changes in a Critical Reactor

“Two group” reactor dynamics theory (see the supplement to Ch. 8) represents the delayed neutrons by a single group. For a *critical* reactor ( $k_i = 1$ ), with initial *steady state* power  $P_0$ , the power as a function of time,  $P(t)$ , after an addition of a step reactivity making  $k_f > 1$ , is:

$$\frac{P(t)}{P_0} = e^{\frac{t}{\tau}} \times \left[ 1 - \frac{\rho_f}{\beta} \right]^{-1} \times \left\{ 1 - \left( \frac{\rho_f}{\beta} \right) e^{-\frac{t}{\tau_R}} \right\} \quad \text{or, rearranging}$$

$$\frac{P(t)}{P_0} = \frac{\left[ e^{\frac{t}{\tau}} - \left( \frac{\rho_f}{\beta} \right) e^{-\frac{t}{\tau_C}} \right]}{\left[ 1 - \frac{\rho_f}{\beta} \right]} \quad \text{or, ignoring the smoothing factor \{xx\}}$$

$$\frac{P(t)}{P_0} \approx e^{\frac{t}{\tau}} \times \left[ 1 - \frac{\rho_f}{\beta} \right]^{-1}$$

where  $\rho = \frac{k-1}{k} = 1 - \frac{1}{k}$  with  $\tau = \frac{1 - \left( \frac{\rho_f}{\beta} \right)}{\lambda \left( \frac{\rho_f}{\beta} \right)}$ ,  $\tau_C = \frac{\left( \frac{\ell}{\beta} \right)}{\left( 1 - \frac{\rho_f}{\beta} \right)}$ , and  $\frac{1}{\tau_R} = \frac{1}{\tau_C} + \frac{1}{\tau}$

This equation is a good approximation if the final reactivity,  $\rho_f$ , is not close to the value of  $\beta$ .<sup>5</sup>

The reactor period,  $\tau$ , is normally much larger than  $\tau_C$ , except when  $\rho \approx \beta$ ; but the equation is not valid for  $\rho \approx \beta$ . As a result, there is no significant error in taking the “rise time parameter”  $\tau_R$  equal to the transient time constant,  $\tau_C$ .

Figures 2 to 5 assume the delayed neutron fraction,  $\beta = 0.00582$ , the prompt neutron lifetime,  $\ell = 0.88$  ms, and the decay constant,  $\lambda = 0.076 \text{ s}^{-1}$ .  $\lambda$  is the inverse of a weighted mean delayed neutron lifetime of 13.2 seconds.

<sup>5</sup> The expression is true both for  $\rho_f \gg \beta$  and  $\rho_f \ll \beta$ . For  $\rho_f \gg \beta$ , the “transient” time constant is positive ( $\tau$  becomes negative). This causes the transient prompt jump to grow rapidly rather than limit its rise.

## Power Changes in a Subcritical Reactor

For an initially subcritical reactor which remains in the subcritical state after a reactivity addition (i.e.  $k_f < 1$  and  $k_i < 1$ ), using notation consistent with that above,  $P(t)$  is:

$$\frac{P(t)}{P_0} = \left( \frac{\rho_i}{\rho_f} \right) - \left( \frac{\rho_i}{\rho_f} - 1 \right) e^{\frac{t}{\tau}} \left[ 1 - \frac{\rho_f}{\beta} \right]^{-1} \left\{ 1 - \left( \frac{\rho_f}{\beta} \right) e^{-\frac{t}{\tau_R}} \right\} \quad \text{where} \quad P_0 = \frac{P_{source}}{-\rho_i} \quad \text{or}$$

$$\frac{P(t)}{P_0} = \left( \frac{\rho_i}{\rho_f} \right) - \left( \frac{\rho_i}{\rho_f} - 1 \right) \times \frac{\left[ e^{\frac{t}{\tau}} - \left( \frac{\rho_f}{\beta} \right) e^{-\frac{t}{\tau_c}} \right]}{\left[ 1 - \frac{\rho_f}{\beta} \right]} \quad \text{or, ignoring the smoothing factor}$$

{xx},

$$P(t) \approx \frac{P_{source}}{(-\rho_f)} \left[ 1 - \frac{\left( 1 - \frac{\rho_f}{\rho_i} \right) e^{\frac{t}{\tau}}}{\left( 1 - \frac{\rho_f}{\beta} \right)} \right]$$

In the first of these, for  $t = 0$ ,  $P(t = 0) = P_0$ , as it should, and when  $t \rightarrow \infty$ , ( $\tau$  is negative),  $P(t \rightarrow \infty)/P_0 = \rho_i / \rho_f$  or  $P(t \rightarrow \infty) = -[P_{source}/\rho_f]$ , also as one expects.

The curly bracket has a value of 1 less than one second after the step reactivity insertion, and corresponds to the prompt jump smoothing term for the critical reactor. Setting this bracket equal to one gives the approximation above for the subcritical reactor. For the subcritical reactor,  $\tau$  is negative, so the large square bracket approaches 1 when  $t \gg |\tau|$ . The overall rise time is governed by the reactor period,  $\tau$ .

How good are these calculated curves in representing the response of a real reactor?

- ∞ The calculations assume the delayed neutrons all have the same half life , 9.1 s. They actually have half lives ranging from 0.2 s to 54.2 s. Accounting for this range, power initially rises faster and further because of the 75% of delayed neutrons with half lives shorter than 9.1 s. Then the rising curve turns over and the longer term power rise, governed by the groups with half lives greater than 9.1 s, approaches equilibrium more gradually.
- ∞ These subtle effects are masked by the more significant operational fact that step increases in reactivity are not possible. Because there is no step increase the prompt jump disappears (so power starts to rise at the rate of the slow reactivity ramp) and the real core overall response lags slightly behind the theoretical curves.